Singularity Consistent Trajectory Control of Y-Axis Robots

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1 Introduction

This article addresses an approach to continuous path planning in the task/tool space in the presence of singularities. In particular it applies to Genmark's Y-axis robots from the GB7Y and GB8Y family. In order to guarantee good enough performance of these robots, the controlled path should be sigularity free, i.e. the "distance" to singularity should be big enough to comply with the limitations imposed on the motor velocities. Regardless of the fact that the users of GB7Y and GB8Y robots are fully aware of the singularity avoidance "rule of thumb", stating that "Stations should be made either perfectly radial or inherently non-radial requiring at least 2" lateral offset from the center of the robot", sometimes this rule is not followed due to the geometrical constraints imposed on the tool-design. In such cases, customers are forced to reduce the velocities of the robot in order to comply with the limited motor velocities and as a result of this the overall performance of the robot deteriorates significantly.

In order to overcome the problem stated above, the proposed approach utilizes the redundancy provided by the Y-axis in order to make the end-effector pole perfectly follow the prescribed straight-line path while at the same time the arm goes through overlap, without reducing the R-axis velocity. The orientation of the end-effector changes in order to guarantee that its pole will lie on the straight-line (longitudinal axis of the station) as the arm extends/retracts.

2 Motion Planning Algorithm

2.1 Synchronization of R and Y axes

The mathematical foundations of the proposed approach are briefly described below. Let us denote the radius-vector of the origin of the straight-line by $\mathbf{r}_0 = [x_0\ y_0]^T$, its unit-vector by $\mathbf{e} = [e_x\ e_y]^T = [\sin(\gamma_0)\ \ -\cos(\gamma_0)]^T$ and the end-effector unit-vector by $\mathbf{e}_\gamma = [\sin(\gamma)\ \cos(\gamma)]^T$. Given R and $\dot{R}$ we would like to calculate $\gamma(R)$ and $\dot{\gamma}(\gamma, R, \dot{R})$ in order to synchronize the motion of γ and R axes so the pole of the end-effector move in a straight-line while the arm goes through overlap.

2.1.1 Synchronization at position level

In order to synchronize the Y axis with R axis, we need to calculate the desired position of γ for given desired position of R. To this end we establish the following relationship between γ and R:

$$\mathbf{r}_0 + S\mathbf{e} = R\mathbf{e}_\theta + E\mathbf{e}_\gamma \tag{1}$$

The last equation is equivalent to

$$(R\mathbf{e}_\theta - \mathbf{r}_0 + S\mathbf{e})^2 - E^2 = 0 \tag{2}$$

which means that the point $\mathbf{r}_0 + S\mathbf{e}$ lies on a circle with a center at $R\mathbf{e}_\theta$, and radius E.

Eq. 2 is a quadratic equation w.r.t. S, which can be effectively resolved if represented in a local coordinate frame with a center at the beginning of the motion-path and an abscise-axis defined by $\mathbf{e}$, see Figure 2.1.2. Let $\mathbf{e}_h$ be an orthogonal vector to $\mathbf{e}$ such that the pair $(\mathbf{e}, \mathbf{e}_h)$ form an orthogonal right-handed basis in $\mathbb{R}^2$. Then the equation of the circle represented in $O_0\mathbf{e}\mathbf{e}_h$ becomes

$$(\mathbf{e}.(\mathbf{r}_c - \mathbf{r}_0) - S)^2 + (\mathbf{e}_h.(\mathbf{r}_c - \mathbf{r}_0))^2 = E^2 \tag{3}$$

The last equation has two solutions for S

$$S_{12} = \mathbf{e}.(\mathbf{r}_c - \mathbf{r}_0) \pm \sqrt{E^2 - (\mathbf{e}_h.(\mathbf{r}_c - \mathbf{r}_0))^2} \tag{4}$$

from which we select the positive one and then solve eq. 1 for γ :

$$\gamma = \arctan \frac{x_0 + Se_x - R\sin(\theta)}{y_0 + Se_y - R\cos(\theta)} \tag{5}$$

2.1.2 Synchronization at velocity level

In order to implement resolved motion velocity control (RMVC) algorithm we need $\dot{\gamma}(\gamma, R, \dot{R})$, which can be obtained via differentiatiing of Eq. 1:

$$\dot{S}\mathbf{e} = \dot{R}\mathbf{e}_\theta + E\dot{\mathbf{e}}_\gamma = \dot{R}\mathbf{e}_\theta + E\frac{d\mathbf{e}_\gamma}{d\gamma}\dot{\gamma} \tag{6}$$

The equation

$$\dot{R}\mathbf{e}_\theta = \dot{S}\mathbf{e} - E\frac{d\mathbf{e}_\gamma}{d\gamma}\dot{\gamma}$$

can be represented in a matrix form

$$\begin{bmatrix} \mathbf{e} & -E\frac{d\mathbf{e}_\gamma}{d\gamma} \end{bmatrix} \begin{bmatrix} \dot{S} \\ \dot{\gamma} \end{bmatrix} = \dot{R}\mathbf{e}_\theta$$

and solved for $\begin{bmatrix} \dot{S} \\ \dot{\gamma} \end{bmatrix}$:

$$\begin{bmatrix} \dot{S} \\ \dot{\gamma} \end{bmatrix} = \left[\mathbf{e} \quad -E\frac{d\mathbf{e}_\gamma}{d\gamma}\right]^{-1} \mathbf{e}_\theta \dot{R} = \frac{\dot{R}}{Det\left[\mathbf{e} \quad -E\frac{d\mathbf{e}_\gamma}{d\gamma}\right]} Adj \left[\mathbf{e} \quad -E\frac{d\mathbf{e}_\gamma}{d\gamma}\right] \mathbf{e}_\theta$$

It is important to be noted here, that in order to avoid problems associated with resolving ill-conditioned tasks, the determinant of A should be as far as possible from zero, i.e. the end-effector should not be close to perpendicular to the straigh-line path. This requirement is always met, when the stations are close to radial. Finally for $\dot{S}$ and $\dot{\gamma}$ we get:

$$\begin{bmatrix} \dot{S} \\ \dot{\gamma} \end{bmatrix} = \frac{\dot{R}}{E\left[e_x \sin(\gamma) + e_y \cos(\gamma)\right]} \begin{bmatrix} E\sin(\gamma) & E\cos(\gamma) \\ -e_y & e_x \end{bmatrix} \begin{bmatrix} \sin(\theta) \\ \cos(\theta) \end{bmatrix}$$

3 Experimental Results

A GB8YPE robot was used in the simulation experiments. Trajectory (blended) moves were defined between three stations A, B and C with the following parameters:

Station A:
- Type: Near-radial
- Pickup coordinates: -17813, 185651, 0, 20000, 37, 267
- Cassette depth: 0
- Retract positions: R = -60000; Y = 10807

Station B:
- Type: Inherently non-radial
- Pickup coordinates: 45000, 250000, 0, 50000, 36, 266
- Cassette depth: 140000
- Retract positions: R = -60000; Y = 10807

Station C:
- Type: Near-radial
- Pickup coordinates: 17803, 185506, 0, 30000, 36, 266
- Cassette depth: 0
- Retract positions: R = -60000; Y = 10807

Stations A and C are close to radial but not exactly (near-radial) while Station B is inherently non-radial. The trajectory between Stations A and B has four segments blended together. The first segment is the one of interest, where the pole of the end-effector goes in a straight-line coincident with the longitudinal axis of Station A, See Figure 3. The smoothness and boundnesness of the motor and tool positions and velocities are clearly seen in Figures 3 and 3. Figures 3-3 show the characteristics of the motion between two near-radial stations. The straight-line segments are clearly seen in Figure 3.

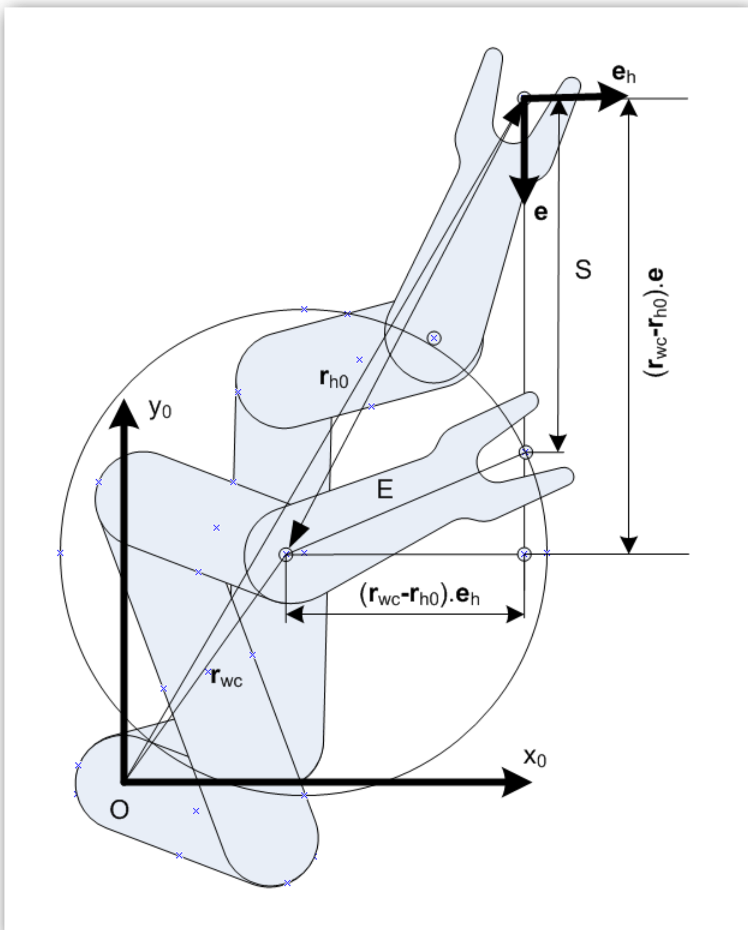

Figure 1: Basic arm geometry

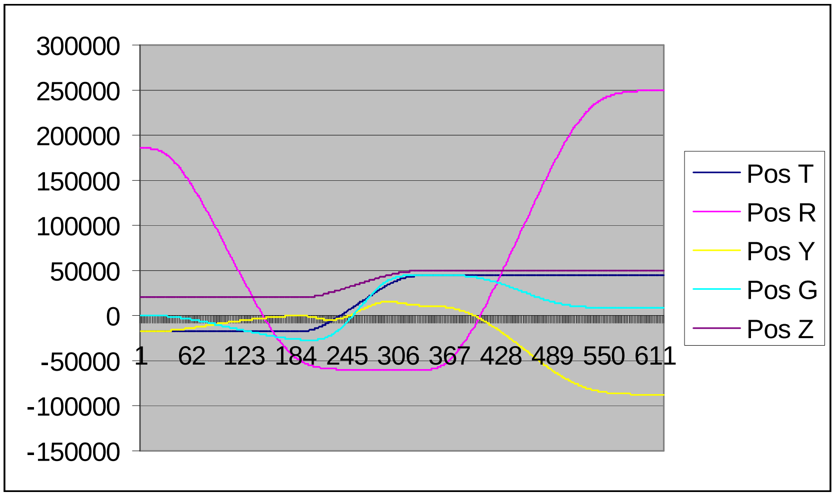

Figure 2: Trajectory move from station A to B: axis posotions

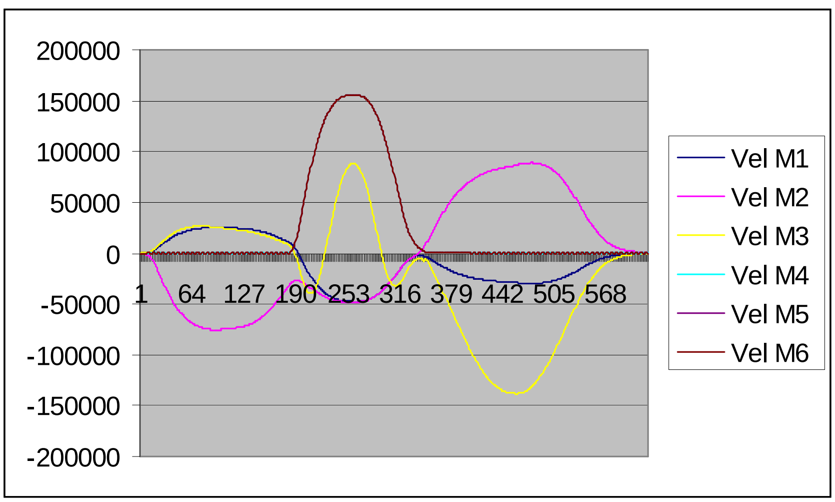

Figure 3: Trajectory move between stations A and B: motor velocities

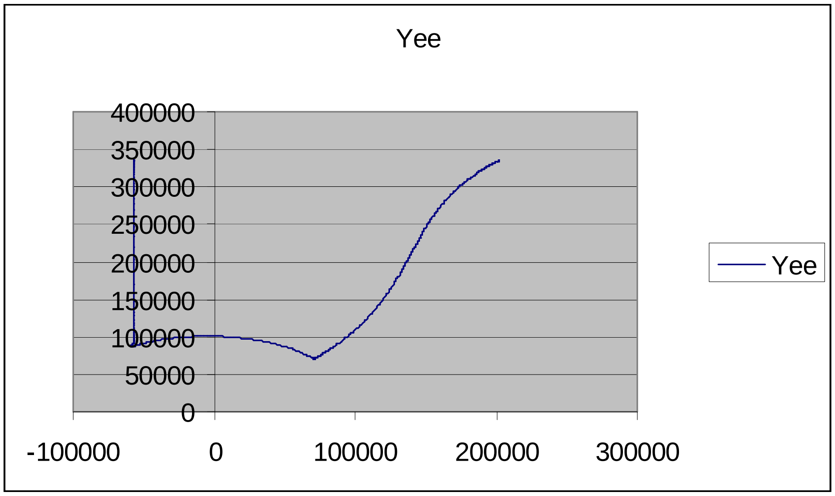

Figure 4: Trajectory move from station A to B: end-effector pole path

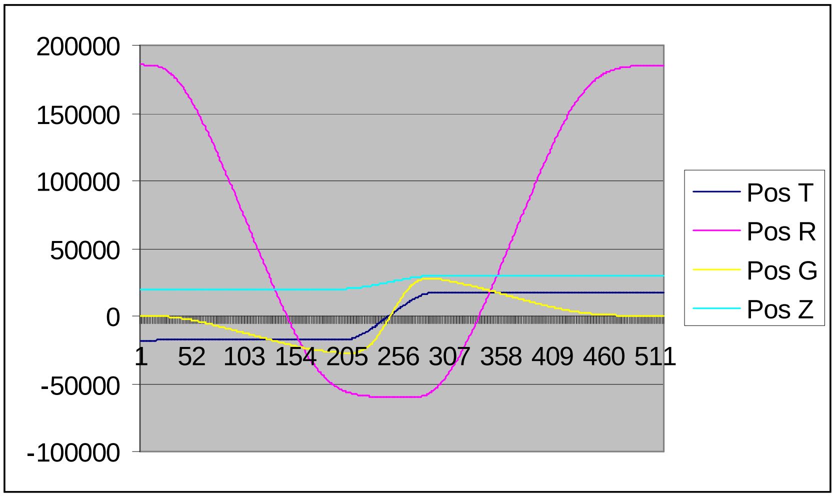

Figure 5: Trajectory move from station A to C: axis posotions

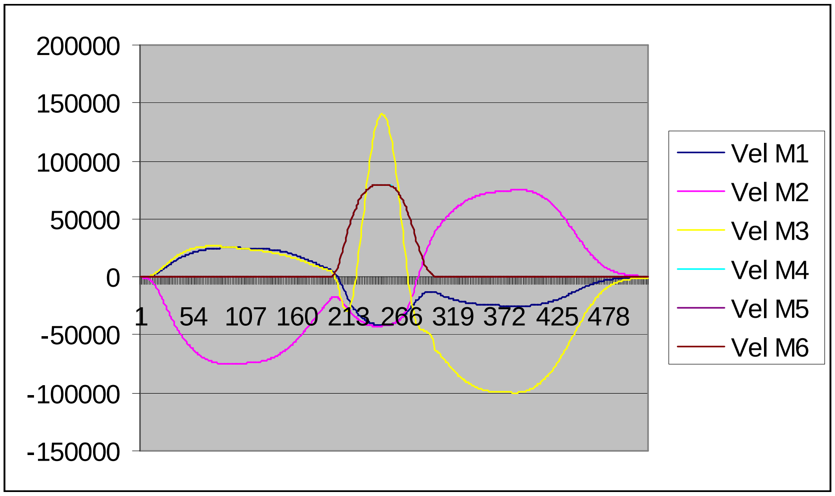

Figure 6: Trajectory move from station A to C: motor velocities

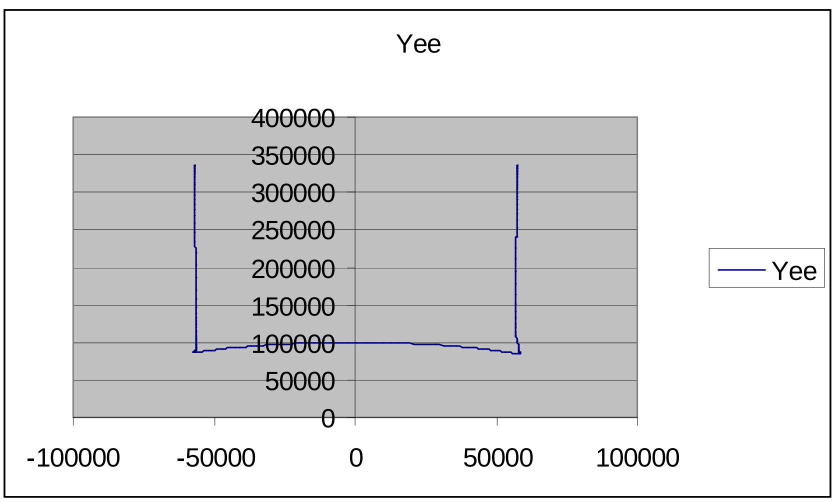

Figure 7: Trajectory move from station A to C: end-effector pole path