# Kinematics of AVR 3000 Vacuum Robot

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April 25, 2011

**Abstract**

The document provides technical data and basic kinematic transformations for AVR 3000. Direct and inverse kinematic problems are solved at position, velocity and acceleration level between three spaces: motor, joint and tool.

## 1 Introduction

The AVR 3000 robot is a 3 degree of freedom (DOF) mechanism, which state (position, velocity and acceleration) typically represented in three different spaces: motor, joint and tool. The state variables and their derivatives, representing the position, velocity and acceleration of the robot at each of these spaces are:

- Motor Space: motor coordinates $\theta_1$, $\theta_2$, $\theta_3$, motor velocities $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$ and motor accelerations $\ddot{\theta}_1$, $\ddot{\theta}_2$, $\ddot{\theta}_3$. The dimensions of the motor positions, velocities and accelerations are [encoder counts], [encoder counts / sec.] and [encoder counts / sec.$^2$]

- Joint Space: joint coordinates $q_1$, $q_2$, $q_3$, joint velocities $\dot{q}_1$, $\dot{q}_2$, $\dot{q}_3$, and joint accelerations $\ddot{q}_1$, $\ddot{q}_2$, $\ddot{q}_3$. The dimensions are: $q_1$ [radians], $q_2$ [radians], $q_3$ [inches], $\dot{q}_1$[radians / sec.], $\dot{q}_2$ [radians / sec.], $\dot{q}_3$ [inches / sec.], $\ddot{q}_1$[radians / sec.$^2$], $\ddot{q}_2$ [radians / sec.$^2$], $\ddot{q}_3$ [inches / sec.$^2$],

- Tool Space: tool coordinates $T$, $R$, $Z$, tool velocities $\dot{T}$, $\dot{R}$, $\dot{Z}$, and tool accelerations $\ddot{T}$, $\ddot{R}$, $\ddot{Z}$. The dimensions are: $T$ [radians], $R$ [inches], $Z$ [inches], $\dot{T}$ [radians / sec.], $\dot{R}$ [inches / sec.], $\dot{Z}$ [inches / sec.], $\ddot{T}$ [radians / sec.$^2$], $\ddot{R}$ [inches / sec.$^2$], $\ddot{Z}$ [inches / sec.$^2$].

Figure 1 shows a simplified kinematic scheme of the AVR 3000 arm.

The arm of AVR 3000 consists of two links with equal length $a$. All kinematic transformations at velocity and acceleration level are linear. The only nonlinear transformations are the direct and inverse kinematics at position level between joint and tool space.

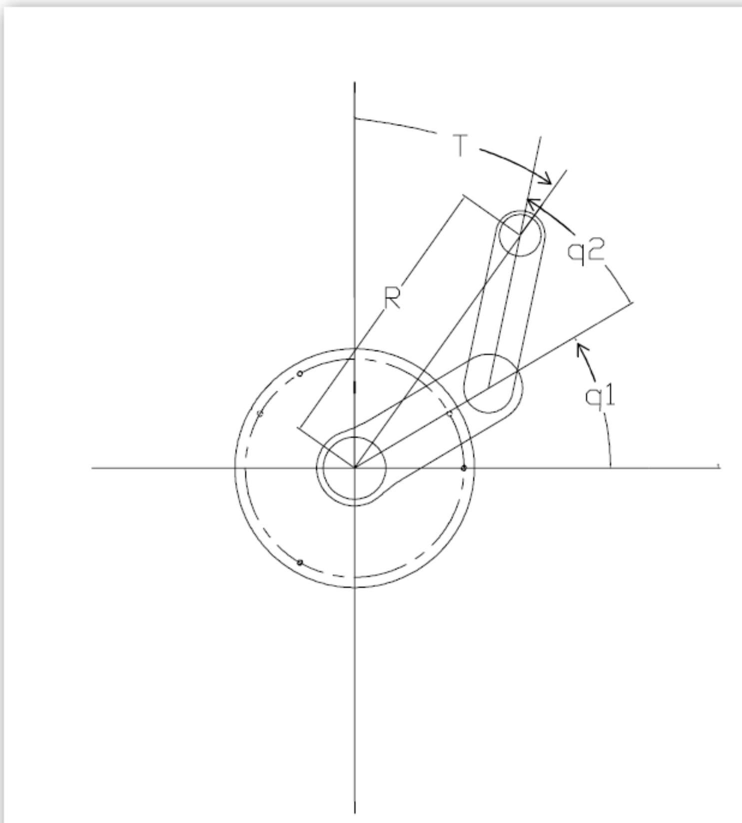


Figure 1: Simplified Kinematics of AVR 3000

# 2 Direct Kinematics

## 2.1 From Motor to Joint Space

### 2.1.1 Position

The joint coordinates can be derived from the motor coordinates as

$$\begin{aligned} q_1 &= J_{11}\theta_1 \\ q_2 &= \pi + J_{21}\theta_1 + J_{22}\theta_2 \\ q_3 &= J_{33}\theta_3 \end{aligned} \tag{1}$$

where $J_{11}$, $J_{21,}$ $J_{22}$ and $J_{33}$ are the non-zero elements of the Jacobian matrix $\mathbf{J}$, that represents the linear relationship between the vector of the motor coordinates $\theta = [\theta_1,\, \theta_2,\, \theta_3]^T$ and the vector of the joint coordinates $\mathbf{q} = [q_1,\, q_2,\, q_3]^T$. We mentioned the matrix representation just for reference (compliance) to commonly accepted notions in the robotic literature and will continue with the scalar representation for simplicity.

The values of the non-zero components of the Jacobian matrix for the specific AVR 3000 geometry and encoder resolution (T and R axes of 2048 e.c./rev and Z axis of 2500 e.c./rev) are:

$$\begin{aligned} J_{11} &= -4.0175691732.10^{-5} \\ J_{21} &= 2.6036376953.10^{-5} \\ J_{22} &= -2.6036376953.10^{-5} \\ J_{33} &= 6.5616798.10^{-6} \end{aligned}$$

### 2.1.2 Velocity

By differentiating 1 we obtain the relationship between the joint velocities and the motor velocities

$$\begin{aligned} \dot{q}_1 &= J_{11}\dot{\theta}_1 \\ \dot{q}_2 &= J_{21}\dot{\theta}_1 + J_{22}\dot{\theta}_2 \\ \dot{q}_3 &= J_{33}\dot{\theta}_3 \end{aligned} \tag{2}$$

The maximum motor velocities are $\dot{\theta}_{1\,max} = \dot{\theta}_{2\,max} = 4500$ RPM $= 471.23$ rad/s and $\dot{\theta}_{3\,max} = 6000$ RPM $= 628.32$ rad/s

### 2.1.3 Acceleration

Next we find the joint accelerations as

$$
\begin{aligned}
\ddot{q}_1 &= J_{11}\ddot{\theta}_1 \\
\ddot{q}_2 &= J_{21}\ddot{\theta}_1 + J_{22}\ddot{\theta}_2 \\
\ddot{q}_3 &= J_{33}\ddot{\theta}_3
\end{aligned} \tag{3}
$$

## 2.2 From Joint to Tool Space

The relationships between the joint and tool coordinates, velocities and accelerations are as follows:

### 2.2.1 Position

$$
\begin{aligned}
T &= \frac{\pi}{2} - q_1 - 0.5q_2 \\
R &= 2a\cos\left(\frac{q_2}{2}\right) \\
Z &= q_3
\end{aligned} \tag{4}
$$

### 2.2.2 Velocity

$$
\begin{aligned}
\dot{T} &= -\dot{q}_1 - 0.5\dot{q}_2 \\
\dot{R} &= -a\sin\left(\frac{q_2}{2}\right)\dot{q}_2 \\
\dot{Z} &= \dot{q}_3
\end{aligned} \tag{5}
$$

### 2.2.3 Acceleration

$$
\begin{aligned}
\ddot{T} &= -\ddot{q}_1 - 0.5\ddot{q}_2 \\
\ddot{R} &= -0.5a\cos\left(\frac{q_2}{2}\right)\dot{q}_2^2 - a\sin\left(\frac{q_2}{2}\right)\ddot{q}_2 \\
\ddot{Z} &= \ddot{q}_3
\end{aligned} \tag{6}
$$

## 2.3 From Joint to Task (Cartesian) Space

### 2.3.1 Position

Let us denote the length of the end-effector by $e$.Then the Cartesian coordinates of the tip of the end-effector are:

$$
\begin{aligned}
x &= a[\cos(q_1) + \cos(q_1 + q_2)] - e\cos(q_1 + 0.5q_2) \\
y &= a[\sin(q_1) + \sin(q_1 + q_2)] - e\sin(q_1 + 0.5q_2)
\end{aligned} \tag{7}
$$

### 2.3.2 Velocity

$$\begin{aligned}
\dot{x} &= a[-\sin(q_1)\dot{q}_1 - \sin(q_1+q_2)(\dot{q}_1+\dot{q}_2)] \\
&\quad +e\sin(q_1+0.5q_2)(\dot{q}_1+0.5\dot{q}_2) \\
&= [-a\sin(q_1) - a\sin(q_1+q_2) + e\sin(q_1+0.5q_2)]\dot{q}_1 \\
&\quad +[-a\sin(q_1+q_2) + e\sin(q_1+0.5q_2)]\dot{q}_2 \\
&= J^x_{11}\dot{q}_1 + J^x_{12}\dot{q}_2
\end{aligned}$$

$$\begin{aligned}
\dot{y} &= a[\cos(q_1)\dot{q}_1 + \cos(q_1+q_2)(\dot{q}_1+\dot{q}_2)] \\
&\quad -e\cos(q_1+0.5q_2)(\dot{q}_1+0.5\dot{q}_2) \\
&= [a\cos(q_1) + a\cos(q_1+q_2) - e\cos(q_1+0.5q_2)]\dot{q}_1 \\
&\quad +[a\cos(q_1+q_2) - e\cos(q_1+0.5q_2)]\dot{q}_2 \\
&= J^x_{21}\dot{q}_1 + J^x_{22}\dot{q}_2
\end{aligned}$$

where

$$\begin{aligned}
J^x_{11} &= -a[\sin(q_1) + \sin(q_1+q_2)] + e\sin(q_1+0.5q_2) \\
J^x_{12} &= -a\sin(q_1+q_2) + e\sin(q_1+0.5q_2) \\
J^x_{21} &= a[\cos(q_1) + \cos(q_1+q_2)] - e\cos(q_1+0.5q_2) \\
J^x_{22} &= a\cos(q_1+q_2) - e\cos(q_1+0.5q_2)
\end{aligned}$$

# 3 Inverse Kinematics

Opposite to the direct kinematics, the inverse kinematics transforms the tool coordinates, velocities and accelerations down to the motor space going through the joint space:

## 3.1 From Tool to Joint Space

### 3.1.1 Position

$$\begin{aligned}
q_2 &= 2\arccos\left(\frac{R}{2a}\right) \\
q_1 &= \frac{\pi}{2} - 0.5q_2 - T \\
q_3 &= Z
\end{aligned} \tag{8}$$

#### 3.1.2 Velocity

$$
\begin{aligned}
\dot{q}_2 &= \frac{-\dot{R}}{a \sin\left(\frac{q_2}{2}\right)} \qquad (9)\\
\dot{q}_1 &= -0.5\dot{q}_2 - \dot{T}\\
\dot{q}_3 &= \dot{Z}
\end{aligned}
$$

#### 3.1.3 Acceleration

$$
\begin{aligned}
\ddot{q}_2 &= \frac{-0.5a\cos\left(\frac{q_2}{2}\right)\dot{q}_2^2 - \ddot{R}}{a \sin\left(\frac{q_2}{2}\right)} \qquad (10)\\
\ddot{q}_1 &= -0.5\ddot{q}_2 - \ddot{T}\\
\ddot{q}_3 &= \ddot{Z}
\end{aligned}
$$

### 3.2 From Joint to Motor Space

#### 3.2.1 Position

$$
\begin{aligned}
\theta_1 &= \frac{q_1}{J_{11}} \qquad (11)\\
\theta_2 &= \frac{q_2 - \pi - \frac{J_{21}}{J_{11}} q_1}{J_{22}}\\
\theta_3 &= \frac{q_3}{J_{33}}
\end{aligned}
$$

#### 3.2.2 Velocity

$$
\begin{aligned}
\dot{\theta}_1 &= \frac{\dot{q}_1}{J_{11}} \qquad (12)\\
\dot{\theta}_2 &= \frac{\dot{q}_2 - \frac{J_{21}}{J_{11}} \dot{q}_1}{J_{22}}\\
\dot{\theta}_3 &= \frac{\dot{q}_3}{J_{33}}
\end{aligned}
$$

#### 3.2.3 Acceleration

$$
\begin{aligned}
\ddot{\theta}_1 &= \frac{\ddot{q}_1}{J_{11}} \qquad (13)\\
\ddot{\theta}_2 &= \frac{\ddot{q}_2 - \frac{J_{21}}{J_{11}} \ddot{q}_1}{J_{22}}\\
\ddot{\theta}_3 &= \frac{\ddot{q}_3}{J_{33}}
\end{aligned}
$$

# 4 Particular Cases of Axes Moves

The AVR 3000 has coupled arm kinematics, i.e. each of the $T$ and $R$ axes moves involve both motors $M_1$ and $M_{2.}$ In other words, there are two electronic gears between the motors $M_1$ and $M_2$, with gear ratios $GearT$ and $GearR$ that result in pure $T$ and $R$ axis motion, respectively.program

## 4.1 T-axis only

Let us consider the direct kinematics equation 5 and substitute the joint velocities in it. As a result we obtain the direct kinematics at velocity level from motor to tool space:

$$\begin{aligned} \dot{T} &= -J_{11}\dot{\theta}_1 - 0.5(J_{21}\dot{\theta}_1 + J_{22}\dot{\theta}_2) = (-J_{11} - 0.5J_{21})\dot{\theta}_1 - 0.5J_{22}\dot{\theta}_2 \quad (14) \\ \dot{R} &= -a\sin\left(\frac{q_2}{2}\right)(J_{21}\dot{\theta}_1 + J_{22}\dot{\theta}_2) \\ \dot{Z} &= J_{33}\dot{\theta}_3 \end{aligned}$$

In order to have a $T$-axis motion only, the $R$-axis velocity must be zero, which is the same as

$$J_{21}\dot{\theta}_1 + J_{22}\dot{\theta}_2 = 0$$

or

$$GearT = \frac{\dot{\theta}_1}{\dot{\theta}_2} = -\frac{J_{22}}{J_{21}} = -\frac{-1.1634.10^{-5}}{1.1634.10^{-5}} = 1$$

The last equation means that in order to perform a pure T-axis motion both $M_1$ and $M_2$ must move in same direction with equal velocities.

## 4.2 R-axis only

A pure $R$-axis motion requires $\dot{T} = 0$, which is the same as

$$(-J_{11} - 0.5J_{21})\dot{\theta}_1 - 0.5J_{22}\dot{\theta}_2 = 0$$

or

$$GearR = \frac{\dot{\theta}_1}{\dot{\theta}_2} = \frac{0.5J_{22}}{-J_{11} - 0.5J_{21}} = \frac{-0.5\,.\,1.1634.10^{-5}}{1.795196.10^{-5} - 0.5\,.\,1.1634.10^{-5}} = 0.47935881$$

# 5 Technical Data

- DOF - 3
- Number of links - 2 + end-effector
- Link lengths - 10.5”

- Range of motion
  - R-axis: -8.9” to 21”
  - T-axis: 0 - 500 deg. (the zero position can be calibrated by the user)
  - Z-axis: 0 - 6”
- Encoder resolution
  - R-axis: 2048 e.c./rev
  - T-axis: 2048 e.c./rev
  - Z-axis: 2500 e.c./rev
- Repeatability
  - R-axis: 0.001”
  - T-axis: 0.01 deg.
  - Z-axis: 0.001”
- Payload - 10 lbs
- Maximum velocities and accelerations:
  - T-axis motor: 4500 RPM at 160 VDC
  - T-axis motor: 4500 RPM at 160 VDC
  - Z-axis motor: 6000 RPM at 60 VDC

the maximum velocities in the tool space (T, R, Z) and in the Cartesian space (X, Y, Z), can be easily calculated based on the positions, maximum velocities and accelerations of the respective motors. The maximum velocities and the accelerations that can be effectively achieved in specific applications depend on the particular operational conditions including loading, control hardware, control scheme, etc. Therefore, once the effective maximum motor velocities and accelerations become known, one can distribute them to the tool and the Cartesian space by solving the Direct Kinematics (see the Matlab code provided).

## 5.1 Basic Geometry

Basic geometrical parameters of the AVR 3000 are provided in Figure 2

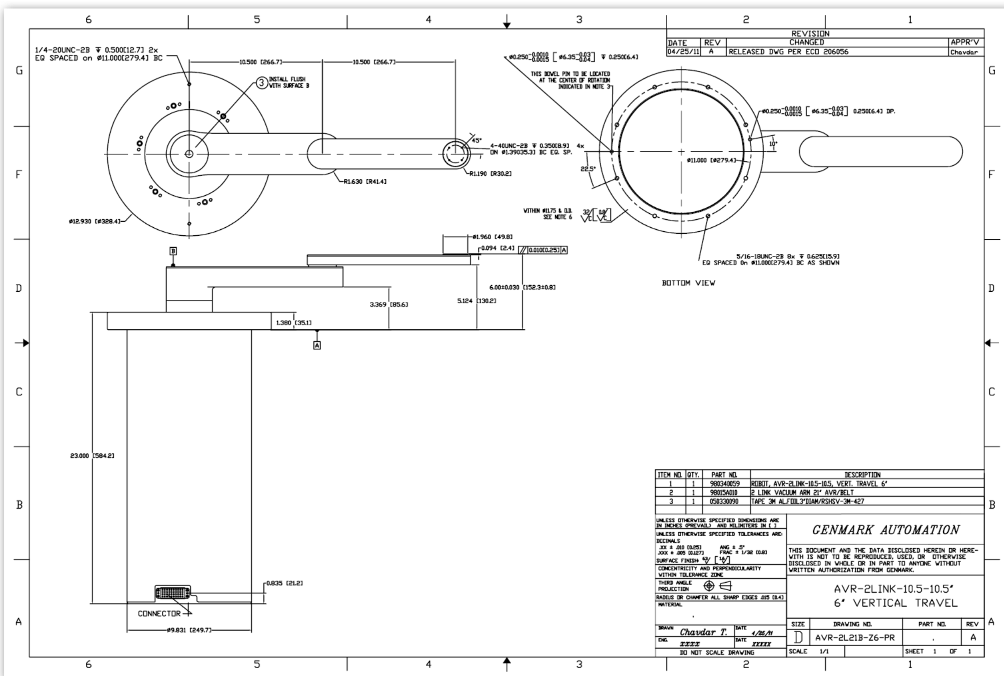


Figure 2: Basic Dimensions of AVR 3000

## 6 Matlab Code

```
%
%Kinematics of AVR 3000 Vacuum Robot
%
EncoderResolutionTR = 2048 * 4;
EncoderResolutionZ = 2500 * 4;
ECPR_TR = EncoderResolutionTR / (2.0 * pi); % Encoder counts per revolution
ECPR_Z = EncoderResolutionZ / (2.0 * pi); % Encoder counts per revolution
IPM = 1000.0 / 25.4; % Inches per meter
GearT = -4.0175691732E-5; % Gear ratio of T-axis transmission [rad/e.c.]
GearR = -2.6036376953E-5; % Gear ratio of R-axis transmission [inch/e.c]
GearZ = 6.5616798E-06; % Gear ratio of Z-axis transmission [inch/e.c]
ArmLength = 21.0 / 2.0; % Lenght of the arm [inch]
Theta = 0.0; % Position of T-axis
MaxMotorVel_T = 4500 * 2 * pi / 60; % Maximum motor velocity in rad/s
MaxMotorVel_R = 4500 * 2 * pi / 60; % Maximum motor velocity in rad/s
MaxMotorVel_Z = 5500 * 2 * pi / 60; % Maximum motor velocity in rad/s
%
% Motion in R-axis direction only
%
MaxMotorVel = [0.479358811234781 * MaxMotorVel_T -MaxMotorVel_R 0]';
%
% Motion in T-axis direction only
%
MaxMotorVel = [MaxMotorVel_T MaxMotorVel_R 0]';
%
% Motion in Z-axis direction only
%
MaxMotorVel = [0 0 MaxMotorVel_Z]';
R = 18.3;
q2 = 2.0 * acos (R / (ArmLength * 2.0))
R = 2.0 * ArmLength * cos (q2 / 2.0)
q1 = (pi - q2) / 2.0
x = ArmLength * (cos (q1) + cos (q1 + q2))
y = ArmLength * (sin (q1) + sin (q1 + q2))
%
% Jacobian matrix (Motor to Joint Space)
%
JMJ(1,1) = -GearT * ECPR_TR;
JMJ(1,2) = 0.0 ;
JMJ(1,3) = 0.0;
```

```
JMJ(2,1) = GearR * ECPR_TR;
JMJ(2,2) = -GearR * ECPR_TR;
JMJ(2,3) = 0.0;
JMJ(3,1) = 0.0;
JMJ(3,2) = 0.0;
JMJ(3,3) = GearZ * ECPR_Z;
%
% Jacobian matrix (Joint to Tool Space)
%
JJT(1,1) = -1.0;
JJT(1,2) = -0.5;
JJT(1,3) = 0.0;
JJT(2,1) = 0.0;
JJT(2,2) = -ArmLength * sin (0.5 * q2);
JJT(2,3) = 0.0;
JJT(3,1) = 0.0;
JJT(3,2) = 0.0;
JJT(3,3) = 1.0;
%
% Jacobian matrix (Tool to Cartesian Space)
%
JX(1,1) = R * cos (Theta);
JX(1,2) = sin (Theta);
JX(1,3) = 0.0;
JX(2,1) = -R * sin (Theta);
JX(2,2) = cos (Theta);
JX(2,3) = 0.0;
JX(3,1) = 0.0;
JX(3,2) = 0.0;
JX(3,3) = 1.0;
%
% Jacobian matrix (Joint to Cartesian Space)
%
JJC(1,1) = -sin(q1) - sin (q1 + q2);
JJC(1,2) = -sin(q1 + q2); MaxTool
JJC(1,3) = 0.0; JJC(2,1) = cos (q1) + cos (q1 + q2);
JJC(2,2) = cos (q1 + q2);
JJC(2,3) = 0.0;
JJC(3,1) = 0.0;
JJC(3,2) = 0.0;
JJC(3,3) = 1.0;
%
% Jacobian matrix (Motor to Tool Space)
%
JMT = JJT * JMJ;
%
```

```
% Jacobian matrix (Motor to Cartesian Space)
%
JMC = JJC * JMJ;
%
% Joint Velocit
%ies
%
MaxJointVel = JMJ * MaxMotorVel
%
% Tool Velocities
%
MaxToolVel = JMT * MaxMotorVel
%
% Cartesian Velocities
%
MaxCartVel = JMC * MaxMotorVel
%
%
% Maximum acceleration required to achieve max. velocity (Tool Space)
%
%
%R - axis
%
RMin = -10.0;
RMax = 20.0;
VelMaxR = MaxToolVel(2)
PathR = abs(RMax - RMin);
AccMaxR = 1.5 * VelMaxR * VelMaxR / PathR
TimeAccR = 1.5 * VelMaxR / AccMaxR
%
%T - axis
%
TMin = 0.0;
TMax = 2 * pi;
VelMaxT = MaxToolVel(1)
PathT = abs(TMax - TMin);
AccMaxT = 1.5 * VelMaxT * VelMaxT / PathT
TimeAccT = 1.5 * VelMaxT / AccMaxT
%
%Z - axis
%
ZMin = 0.0;
ZMax = 6.0;
VelMaxZ = MaxToolVel(3)
PathZ = abs(ZMax - ZMin);
AccMaxZ = 1.5 * VelMaxZ * VelMaxZ / PathZ
```

TimeAccZ = 1.5 * VelMaxZ / AccMaxZ