Force Analysis of GB9 Vacuum Robot

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1 Introduction

The goal of this document is to analyze the maximum forces applied by/to the GB9 robot in case of interfering with obstacles in impact or quasi-staic mode. The evaluation is performed by taking into consideration the operational conditions of a specific application such as geometrical constraints, maximum velocities and accelerations, loading, etc. The maximum forces applied by/to the robot are limited by a dedicated hardware current protection system with software set current limits. Once these limits are violated for more than 100 ms, the current protection disables the motors. Several use cases addressing typical scenarios of robot operation, were simulated.

2 Operational Conditions

The operational conditions are specific to the customer application and identified below:

2.1 Parameters:

- Load: 4.5 kg
- Retract Position: -13"
- Maximum Extension used by customer: 18.3"
- Maximum Velocities:
  - R axis: 14.45" (0.367 m/s) reached at extension of the arm at 2.65"
  - T axis: 109.54 deg/s (3.82 rad/s)
- Maximum Accelerations
  - R axis: 10"/s^2
  - T axis: 100 deg/s^2

- End Effector Mass: 1.5 kg
- Manipulated Object Mass: 4.5 kg
- End Effector Length: 18.25"
- Link Length: 12.5"
- Motor Torque Coefficient: 28.13 oz.inch/Amp
- Motor Continuous Torque: 150 oz.inch (1.06 N.m)
- Current for activating the current protection: 5.33 Amp resulting in 150 oz.in (1.06 N.m). This is valid for all motors
- Anticipated time to stop stopping time after impact or abrupt stop - 100 ms (0.1 s)

3 Theoretical Background

There is an inherent duality between the tasks of transforming velocities and forces within a mechanism. Let us denote the Jacobian matrix that transforms the motor velocities into joint velocities by $\mathbf{J}_{mj}$, the joint velocities into end-effector velocities by $\mathbf{J}_{jt}$, and the motor velocities into end-effector velocities by $\mathbf{J}_{mt}$

In the GB9 case

$$\mathbf{J}_{mj} = \begin{pmatrix} n_T & 0 & 0 \\ n_R & -n_R & 0 \\ 0 & 0 & n_Z \end{pmatrix} \tag{1}$$

$$\mathbf{J}_{jt} = motorsof \begin{pmatrix} -sin(q_1) - sin(q_1 + q_2) & -sin(q_1 + q_2) & 0 \\ cos(q_1) + cos(q_1 + q_2) & cos(q_1 + \ddot{R}_{impact} q_2) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

$$\mathbf{J}_{mt} = \mathbf{J}_{jt}\mathbf{J}_{mj} \tag{3}$$

and the motor torques $\mathbf{M}$ are transformed into forces $\mathbf{F}$ acting on the end-effector by using the formula

$$\mathbf{F} = (\mathbf{J}_{mt}^T)^{-1}\mathbf{M} \tag{4}$$

In the equations 1-4, n_T, n_R, n_Z stand for the gear-ratios of the respective axes, $q_1$and q_2- for the joint angles, $\mathbf{F} = (\ F_x \quad F_y \quad F_z \ )^T$ - for the forces acting at the end-point of the last link, and $\mathbf{M} = (\ M_1 \quad M_2 \quad M_3 \ )^T$- for the motor torques. As it is well known, the motor torques are proportional to the motor current, i.e. $M_i = k_t I_i$, where k_t is the torque constant of the motor. The maximum motor current and the maximum motor torque are limited by the hardware current protection, which disables the motor if the maximum current limit is exceeded for more than 100 ms. The motors for all GB9 axes have continuous torque of 150 oz.inch and default setting of the current protection limit to 5.33 Amp. The torque constant for all motors is equal to 28.13 oz.in/Amp, which gives 150 oz.in or 1.06 N.m. Therefore this setting of the current limits allows to fully exersize the continuous torque of the motors. As a matter of fact, the motors have been selected with enough margin to cover most demanding force applications and there is a room for optimization (lowering the current limit) in order to reduce the force applied to the obstacle in case of collision. The calculations below are conducted for motor torques corresponding to 5.33 Amp current, namely

$$\mathbf{M} = (\ 1.06 \quad 1.06 \quad 1.06\ )^T$$

4 Simulation Results

As it is seen from eq. 2 the force transformation is dependent on the configuration of the arm and more precisely on the extension / retraction of the arm specified by the q_2. The force transformation does not depend on q_1, i.e. on the position of T-axis. The following use cases of applying external forces to the robot arm will be considered:

4.1 The R-axis arm moves with maximum velocity

Given the fact that the retract position of the arm inside the minimum envelope is -13" and the maximum extension used by the customer is 18.3", the R-axis reaches its maximum velocity of 14.45 inch/s (0.367 m/s) in the middle of the motion range, corresponding to position of R-axis of 2.65" or to $q_2 = 2.9292$ rad. The other joint angle $q_1$is calculated in such a way ($q_1 = 0.1062$ rad) so the tip of the terminal end of the second link lie on the OY axis of the world coordinate frame OXYZ. The coordinates of the tip are x = 0 y = 0.0673 m. The force **F** is

$$\mathbf{F} = (\ -200.0 \quad -21.01 \quad 399.61\ )^T \tag{5}$$

i.e. $F_x = -200$ N, $F_y = -21.01$N and $F_z = 399.61$N. In other words, these are the external or inertia forces that would trigger the current protection. If the arm collides with an obstacle or is commanded to stop abruptly, and it is assumed that the stopping time is 0.1 sec., the impact acceleration of R-axis will be

$$\ddot{R}_{impact} = \dot{R}_{max}/0.1 = 0.367/0.1\ddot{R}_{impact} = 3.67\, m/s^2$$

The respective impact force acting in OY direction will be

$$F_{y\, impact} = m_{load}\ddot{R}_{impact} = 6\, .\, 3.67 = 22.02\, N$$

As seen from eq. 5 the impact force would activate the current protection. If there is no impact, or the last is negligible, quasi-static forces as defined by eq. 5 will activate the current protection.

4.2 The R axis is with maximum extension

As mentioned above, the maximum extension of the arm for the specific customer application is 18.3". The calculated geometrical and force parameters for this extension are $q_2 = 1.4991$ rad, $q_1 = 0.8213$ rad, $x = 0$, $y = 0.4648$ m and

$$\mathbf{F} = \begin{pmatrix} -28.96 & -30.67 & 399.61 \end{pmatrix}^T$$

It becomes obvious, that compared to the previous example the force in x-direction that would activate the current protection reduce significantly, since the extension of the arm is bigger.

4.3 The R axis is in the minimum envelope

The position of R-axis inside the minimum envelope is 13". The calculated geometrical and force parameters for this extension are $q_2 = 4.2353$ rad, $q_1 = -0.5469$ rad, $x = 0$, $y = -0.3302$ m and

$$\mathbf{F} = \begin{pmatrix} 40.76 & -24.46 & 399.61 \end{pmatrix}^T \tag{6}$$

4.4 The T axis rotates with maximum velocity

In this case, the arm is within the minimum envelope (Pos R = -13") and T-axis rotates with maximum velocity of 3.82 rad/s. If the arm collides with an obstacle or it is forced to stop abruptly with anticipated stopping time of 0.1 sec., the impact acceleration of T-axis would be calculated as follows: when T-axis rotates with angular velocity of 3.81 rad/s the velocity of the tip of the terminal end of the second link will be $x = 0.33 \,.\, 3.82 = 1.26$ m/s. The impact acceleration and force will be

$$\ddot{x}_{impact} = \dot{x}_{max}/0.1 = 1.26/0.1 = 12.6\, m/s^2$$

$$F_{x\, impact} = m_{load}\ddot{x}_{impact} = 6 \,.\, 12.6 = 75.6\, N$$

As seen from eq. 6the impact force will trigger the current protection because it exceeds $F_x = 40.76\, N$