Dynamics of Typical Moves of AVR 3000

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1 Introduction

The goal of this document is to evaluate the loading (static and dynamic) to the supporting bearing of the end-effector (EE) of AVR 3000 in compliance with Oxford Industries specific requirements. The EE is with a custom shape, see Figure 5.2, designed to carry a 330 mm round substrate with a weight of 1.2 kg. The mass of the end-effector and the manipulated substrate is 3.18 kg. The center of mass of the end-effector + substrate is defined by the radius-vector ρ_c with respect to the coordinate frame $Oxyz$ firmly attached to the end-effector with its origin at the center of the substrate. For the sake of simplicity, we assume that the AVR 3000 is in a configuration, where the axes of the coordinate frame $Oxyz$ are parallel to the axes of the world coordinate frame $O_0x_0y_0z_0$. The inertia-matrix of the end-effector + substrate defined with respect to the center of mass and represented in the coordinate frame $Oxyz$ is

$$\mathbf{I}^c = \begin{bmatrix} 0.016 & 0 & 0 \\ 0 & 0.055 & 0 \\ 0 & 0 & 0.07 \end{bmatrix}$$

Since the principal axes of inertia coincide with the axes of the coordinate frame $Oxyz$, the matrix $\mathbf{I}^c$ is diagonal with nonzero elements the principal moments of inertia $I_{xx} = 0.016\,kg.m^2$, $I_{yy} = 0.055\,kg.m^2$ and $I_{zz} = 0.07\,kg.m^2$. The joint supporting the end-effector to the last link of the robot comprises a roller-bearing with radial and axial load-capabilities in the range of 3.2 kN. For the purpose of the dynamic calculations this support is represented by the reaction force vector $\mathbf{F}_r$ and the torque vector $\mathbf{M}_r$. All three components of $\mathbf{F}_r$ are reaction forces in x, y and z directions whereas the first two components of $\mathbf{M}_r$ are reaction moments with respect to the x and y axes and the third component M_z is the active (driving) torque, that maintains the orientation of the end-effector.

The goal of the dynamic study is to determine the vectors $\mathbf{F}_r$ and $\mathbf{M}_r$ for typical moves of the AVR 3000.

2 Typical Moves of AVR 3000

In order to perform the manipulation tasks and to comply with the constraints imposed by a typical tool configuration the AVR 3000 robot executes the following moves:

- Perform rotation around the central -axis (T) and move in the vertical direction (Z) when the robot-arm (R) is in the minimum swinging envelope. The worst case from dynamics standpoint is when the T and Z moves happen simultaneously (synchronously).
- Extends or retracts the arm (R-axis motion) without moving T and Z axes.

In general the robot can move all three axes at one and the same time, or move the T and Z axes when the arm is extended, but this scenario does not comply with the constraints of the majority of the customer tools. The computational scheme presented in this document can address this general case, provided the extension of the arm is defined.

3 Kinematics of the End-Effector + Substrate

In order to derive the dynamics of the end-effector + substrate we need to calculate the following basic kinematic properties:

- acceleration of the center of mass for given axes accelerations

  $$\ddot{r}_c = [\ddot{R} \;\; (R+E)\ddot{T} \;\; \ddot{Z}+g]^T$$

  where R is the position of R-axis, E is the distance from the center of rotation of the end-effector to its center of mass, $\ddot{T}$, $\ddot{R}$ and $\ddot{Z}$ are the axes accelerations, and g is the gravity acceleration.
- angular velocity of the end-effector + substrate $\omega = [0 \;\; 0 \;\; \dot{T}]^T$ where $\dot{T}$ is the T-axis velocity
- angular acceleration of the end-effector + substrate $\dot{\omega} = [0 \;\; 0 \;\; \ddot{T}]^T$

4 Dynamics of the End-Effector + Substrate

The dynamics of the end-effector + substrate is defined by the Newton-Euler equations of motion of a rigid body:

$$\mathbf{F} = -m\ddot{r}_c \tag{1}$$

$$\mathbf{I}^c\dot{\omega} + \omega \times (\mathbf{I}^c.\omega) = \mathbf{M}^c \tag{2}$$

where m is the mass of the end-effector + substrate, F is the general vector of all forces acting on the end-effector + substrate, and $\mathbf{M}^c$ is the torque vector of all forces acting on the end-effector with respect to the center of mass.

Taking into account the already defined reaction forces and torques, we can rewrite 1 and 2 in the form:

$$\mathbf{F} + \mathbf{F}_r = \mathbf{0}$$

$$\mathbf{M}_r + \mathbf{I}^c\dot{\omega} + \omega \times (\mathbf{I}^c.\omega) + \rho \times F = \mathbf{0}$$

which is the same as

$$\mathbf{F}_r = -m\ddot{r}_c \tag{3}$$

$$\mathbf{M}_r + \mathbf{I}^c\dot{\omega} + \omega \times (\mathbf{I}^c.\omega) - \rho \times m\ddot{r}_c = 0 \tag{4}$$

Solving Eq. 4 for $\mathbf{M}_r$ gives:

$$\mathbf{M}_r = \rho \times m\ddot{r}_c - \mathbf{I}^c\dot{\omega} - \omega \times (\mathbf{I}^c.\omega) \tag{5}$$

In the above equation ρ stands for the radius-vector of the tip of the joint between the last link and the end-effector with respect to the center of mass, represented in the coordinate frame $Oxyz$. For the specific geometry of the end-effector this radius-vector is:

$$\rho = [-0.31 \;\; 0 \;\; 0]^T$$

5 Calculations

5.1 T and Z Motion

We will be performing the calculations for the following working velocities and accelerations of the axes:

$\dot{T} = 2.09$ [rad/s], $\dot{R} = 0$, $\dot{Z} = 0.127$[m/s]

$\ddot{T} = 2.62$[rad/s^2], $\ddot{R} = 0$, $\ddot{Z} = 0.305$[m/s^2]

The angular velocity, acceleration of the end-effector+substrate and the acceleration of the center of mass are:

$$\omega = [0 \;\; 0 \;\; 2.09]^T$$

$$\dot{\omega} = [0 \;\; 0 \;\; 2.62]$$

$$\ddot{r}_c = [0 \;\; (-0.155 + 0.311)2.62 \;\; 0.305 + 9.81]^T = [0 \;\; 0.156 \;\; 10.12]^T$$

By substituting the above motion parameters into Eq. 3 and Eq. 5 we obtain:

$$\mathbf{F}_r = -3.18 \begin{bmatrix} 0 \\ 0.156 \\ 10.12 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.496 \\ -32.18 \end{bmatrix} [N]$$

$$\mathbf{M}_r = \begin{bmatrix} -0.31 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0.496 \\ 32.18 \end{bmatrix} - \begin{bmatrix} 0.016 & 0 & 0 \\ 0 & 0.055 & 0 \\ 0 & 0 & 0.07 \end{bmatrix} . \begin{bmatrix} 0 \\ 0 \\ 2.62 \end{bmatrix} -$$

$$- \begin{bmatrix} 0 \\ 0 \\ 2.09 \end{bmatrix} \times \left(\begin{bmatrix} 0.016 & 0 & 0 \\ 0 & 0.055 & 0 \\ 0 & 0 & 0.07 \end{bmatrix} . \begin{bmatrix} 0 \\ 0 \\ 2.09 \end{bmatrix} \right)$$

$$\mathbf{M}_r = \begin{bmatrix} 0 \\ 9.9758 \\ -0.3372 \end{bmatrix} [N.m]$$

As it is seen from the calculations above, the radial and axial loading of the cross-bearing at the end-effector joint are $F_{ry} = -0.496$ [N] and $F_{rz} = -32.18$ [N] respectively. The reactive torque at this is $M_{ry} = 9.9758$ [N.m]

5.2 R-Motion

In this case the velocities and accelerations of T and Z axes are zero:

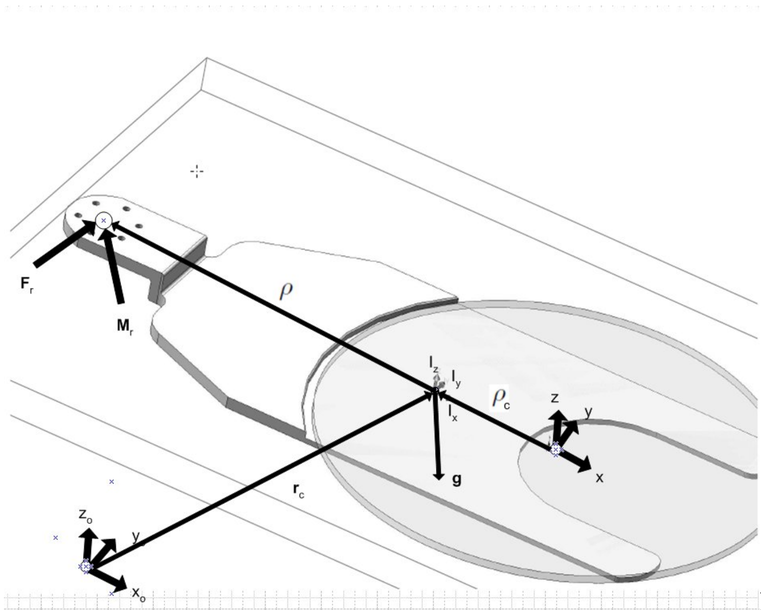
$\dot{T} = 0$, $\dot{R} = 0.305$[m/s], $\dot{Z} = 0$
$\ddot{T} = 0$, $\ddot{R} = 0.381$[m/s^2], $\ddot{Z} = 0$
$\omega = [0\ \ 0\ \ 0]^T$, $\dot{\omega} = [0\ \ 0\ \ 0]^T$
$\ddot{r}_c = [0.381\ \ 0\ \ 9.81]^T$

$$\mathbf{F}_r = -3.18 \begin{bmatrix} 0.381 \\ 0 \\ 9.81 \end{bmatrix} = \begin{bmatrix} -1.2116 \\ 0 \\ -31.19 \end{bmatrix} [N]$$

$$\mathbf{M}_r = \begin{bmatrix} -0.31 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1.2116 \\ 0 \\ 31.19 \end{bmatrix}$$

$$\mathbf{M}_r = \begin{bmatrix} 0 \\ 9.67 \\ 0 \end{bmatrix} [N.m]$$

As it is seen from the calculations above, the radial and axial loading of the cross-bearing at the end-effector joint are $F_{rx} = -1.2116$ [N] and $F_{rz} = -31.19$ [N] respectively. The reactive torques at this joint is $M_{ry} = 9.67$ [N.m]



Properties

Figure 1: End-Effector Properties

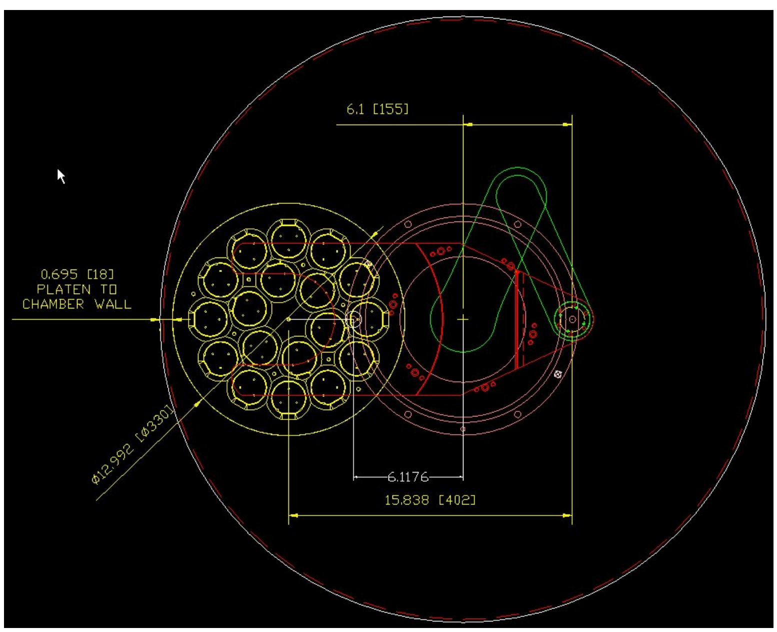


Figure 2: AVR 3000 Inside the Minimum Envelope